Overview of Optimization-based Blind Equalizer Design

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***Abstract*—This paper briefly reviews the blind equalization algorithms that detect the modulated signal without knowledge of channel information. We describe how to design optimization objective and constraint functions for blind equalization.**

***Index Terms*—Blind equalization, symbol detection, optimization problems.**

I. Introduction

We consider blind equalization in inter-symbol interference channels. The L-times over-sampled signal is given by

$$x_k^{(l)} = \sum_{m=0}^{K-1} s_{k-m} h_m^{(l)} + z_k^{(l)},\ l = 1, \cdots, L, \tag{1}$$

where s_k is a transmitted symbol at time k and $\{h_m^{(l)}, m = 0, \cdots, K-1, l = 1, \cdots, L\}$ is an unknown channel vector. For a linear equalization filter $\{\theta_m^{(l)}, m = 0, \cdots, M-1,\ l = 1, \cdots, L\}$, the linear equalized signal is given by

$$y_k = \sum_{l=1}^{L} \sum_{m=0}^{M-1} \theta_m^{(l)} x_{k-m}^{(l)}. \tag{2}$$

The blind detection problem is to design $\{\theta_m^{(l)}\}$ such that the equalized signal y_k recovers the transmitted symbol s_k. There have been many optimization-based algorithms to perform blind detection. In this paper, we briefly review the blind equalization algorithms with an emphasis on different objective and constraint functions.

II. Blind Equalization Algorithms

A. Constant Modulus Algorithm (CMA)

For PSK symbols, the symbols have constant modulus, i.e., $|s_k| = 1$. Thus, CMA aims to find the filter θ to make $|y_k|$ as close as to 1

$$\min_{\theta} J(\theta) = E[(|y_k|^2 - 1)^2]. \tag{3}$$

The stochastic gradient method provides the CMA update [1]

$$\theta^{(n+1)} = \theta^{(n)} - \mu \nabla J(\theta^{(n)}). \tag{4}$$

B. Linear Programming

For QAM symbols, the symbols are confined in box constraints, that is, real and imaginary parts belong to set $\{x| -\tau \le x \le \tau\}$. Since QAM signal constellation has symmetry, the cost function considers only the real part

$$J(\theta) = \max_{k} |Re\{y_k\}|. \tag{5}$$

The linear programming (LP) for the blind equalization [2] is

$$\begin{aligned} \min_{\tau,\theta}\ & \tau \\ \text{s.t. }\ & -\tau \le Re\{y_k\} \le \tau,\ \forall k \\ & \sum_{l=1}^{L} Re\{\theta_0^{(l)}\} + Im\{\theta_0^{(l)}\} = 1, \end{aligned} \tag{6}$$

where the equality constraint is introduced to prevent θ from being trivial zero.

C. Nuclear Norm Minimization

If we define sub-equalizer parameter matrix $\Theta = [a_{ij}]_{M\times L}$, where $a_{ij} = \theta_{i-1}^{(j)}$, then Θ is rank-deficient. Thus, we minimize rank(Θ) to fully utilize the over-sampling. Since the nuclear norm $\|\Theta\|_*$ is the convex relaxation to non-convex rank(Θ), the nuclear norm minimization method [3] is

$$\begin{aligned} \min_{\tau,\theta}\ & \tau + w\|\Theta\|_* \\ \text{s.t. }\ & -\tau \le Re\{y_k\} \le \tau,\ \forall k \\ & \sum_{l=1}^{L} Re\{\theta_0^{(l)}\} + Im\{\theta_0^{(l)}\} = 1. \end{aligned} \tag{7}$$

III. Summary

This paper briefly reviewed the blind equalization algorithms from the perspective of optimization. We can achieve excellent detection performance by designing novel objective and constraint functions that express the properties of discrete signal constellations well.

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